- $\vec{OM}(t) = r\,\vec{u_r} + z\,\vec{u_z} = b e^{-\frac{2t}{\tau}}\,\vec{u_r}$

$\vec{v}(t) = \dfrac{d\vec{OM}(t)}{dt} = -\dfrac{2b}{\tau} e^{-\frac{2t}{\tau}}\,\vec{u_r} + r\omega\,\vec{u_\theta}$

- $OM = \sqrt{r^2(t) + z^2(t)} = b e^{-\frac{2t}{\tau}}$

$v = \sqrt{v_r^2(t) + v_\theta^2(t) + v_z^2(t)} = \sqrt{\dfrac{4b^2}{\tau^2} e^{-\frac{4t}{\tau}} + (r\omega)^2} = \sqrt{\dfrac{4b^2}{\tau^2} e^{-\frac{4t}{\tau}} + b^2 e^{-\frac{4t}{\tau}}\omega^2}$

$v = b e^{-\frac{2t}{\tau}} \sqrt{\dfrac{4}{\tau^2} + \omega^2}$

$\Rightarrow$ $\vec{OM}\cdot\vec{v}$ peut se calculer en

- utilisant les coordonnées et composantes : $\vec{OM}\cdot\vec{v} = r \times \left(-\dfrac{2b}{\tau} e^{-\frac{2t}{\tau}}\right) + 0 \cdot r\omega + z \cdot 0$
- utilisant le cosinus : $\vec{OM}\cdot\vec{v} = OM \times v \times \cos(\vec{OM}, \vec{v})$

$\hookrightarrow -r\dfrac{2b}{\tau} e^{-\frac{2t}{\tau}} = \underbrace{b e^{-\frac{2t}{\tau}}}_{OM} \times \underbrace{b e^{-\frac{2t}{\tau}} \sqrt{\dfrac{4}{\tau^2} + \omega^2}}_{v} \times \cos\theta$

$\Rightarrow \underbrace{-\cancel{b e^{-\frac{2t}{\tau}}}}_{r} \dfrac{2\cancel{b}}{\tau} \cancel{e^{-\frac{2t}{\tau}}} = \cancel{b^2} \cancel{e^{-\frac{4t}{\tau}}} \sqrt{\dfrac{4}{\tau^2} + \omega^2} \times \cos\theta$

$\Rightarrow -\dfrac{2}{\tau} = \sqrt{\dfrac{4}{\tau^2} + \omega^2}\,\cos\theta \Rightarrow \boxed{\cos\theta = \dfrac{-\frac{2}{\tau}}{\sqrt{\dfrac{4}{\tau^2} + \omega^2}}}$